Analysis and Design of Linear Control System –Part2-

Instructor: Prof. Masayuki Fujita

Basic Feedback Loop

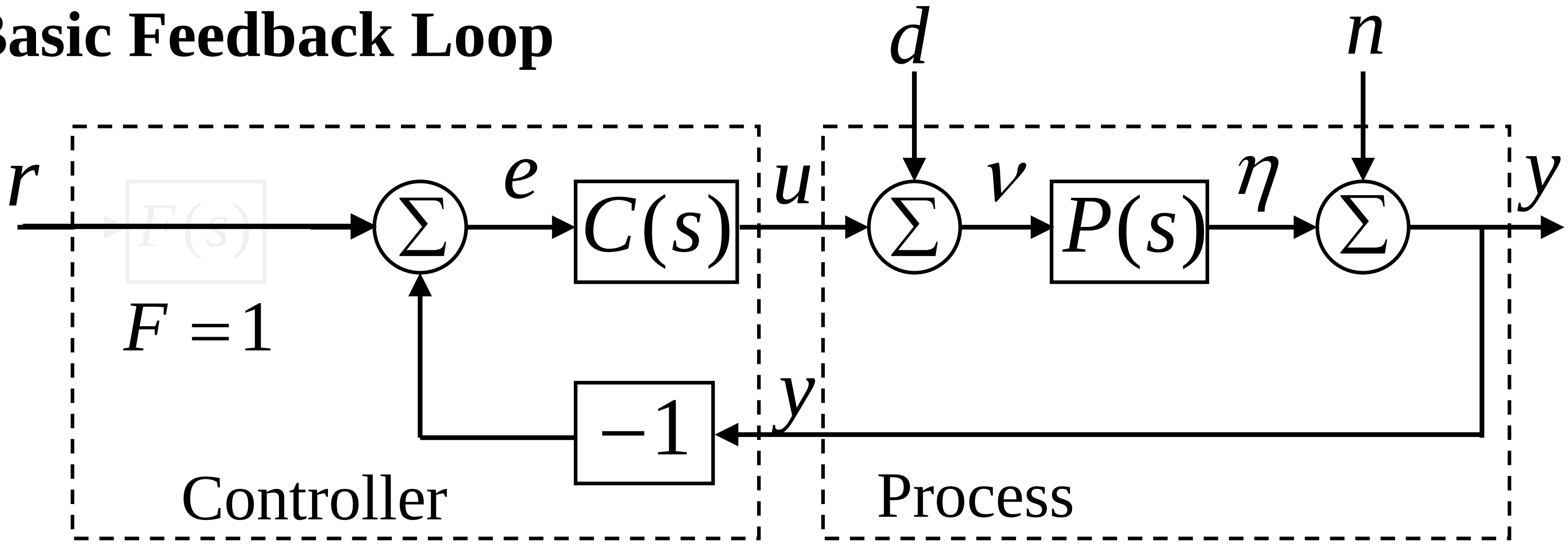


Fig. 11.1

$P(s)$: Process

$C(s)$: Feedback block

($F(s)$: Feedforward block)

r : Reference signal

d : Load disturbance

n : Measurement noise

e : Output error

u : Control variable

η : Process output

y : Measured signal

Control System Transfer Functions

$$y = \eta + n$$
$$= P\nu + n = P(u+d) + n$$
$$= PCe + Pd + n$$
$$= PC(r-y) + Pd + n$$
$$(1+PC)y = PCr + Pd + n \;\Rightarrow\; y = \frac{PC}{1+PC}r + \frac{P}{1+PC}d + \frac{1}{1+PC}n$$

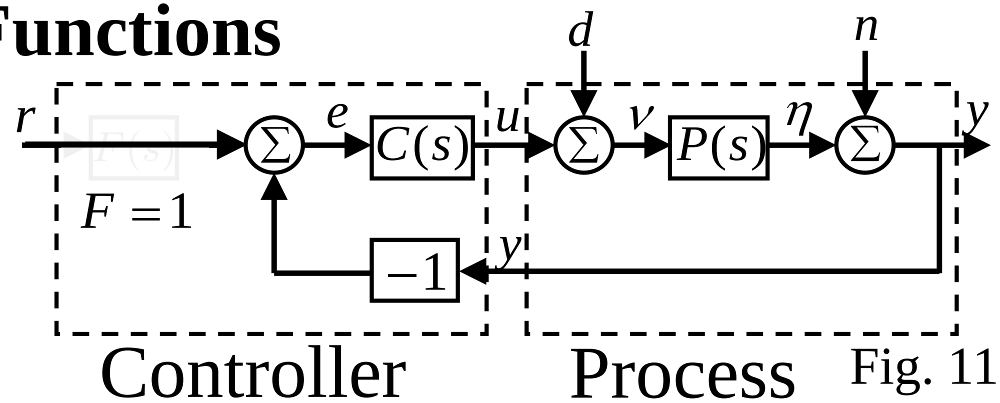


Fig. 11.1

$$\begin{bmatrix} y \\ \eta \\ \nu \\ u \\ e \end{bmatrix} = \begin{bmatrix} \frac{PC}{1+PC} & \frac{P}{1+PC} & \frac{1}{1+PC} \\ \frac{PC}{1+PC} & \frac{P}{1+PC} & \frac{-PC}{1+PC} \\ \frac{C}{1+PC} & \frac{1}{1+PC} & \frac{-C}{1+PC} \\ \frac{C}{1+PC} & \frac{-PC}{1+PC} & \frac{-C}{1+PC} \\ \frac{1}{1+PC} & \frac{-P}{1+PC} & \frac{-1}{1+PC} \end{bmatrix} \begin{bmatrix} r \\ d \\ n \end{bmatrix} \qquad (11.1)$$

Gang of Four

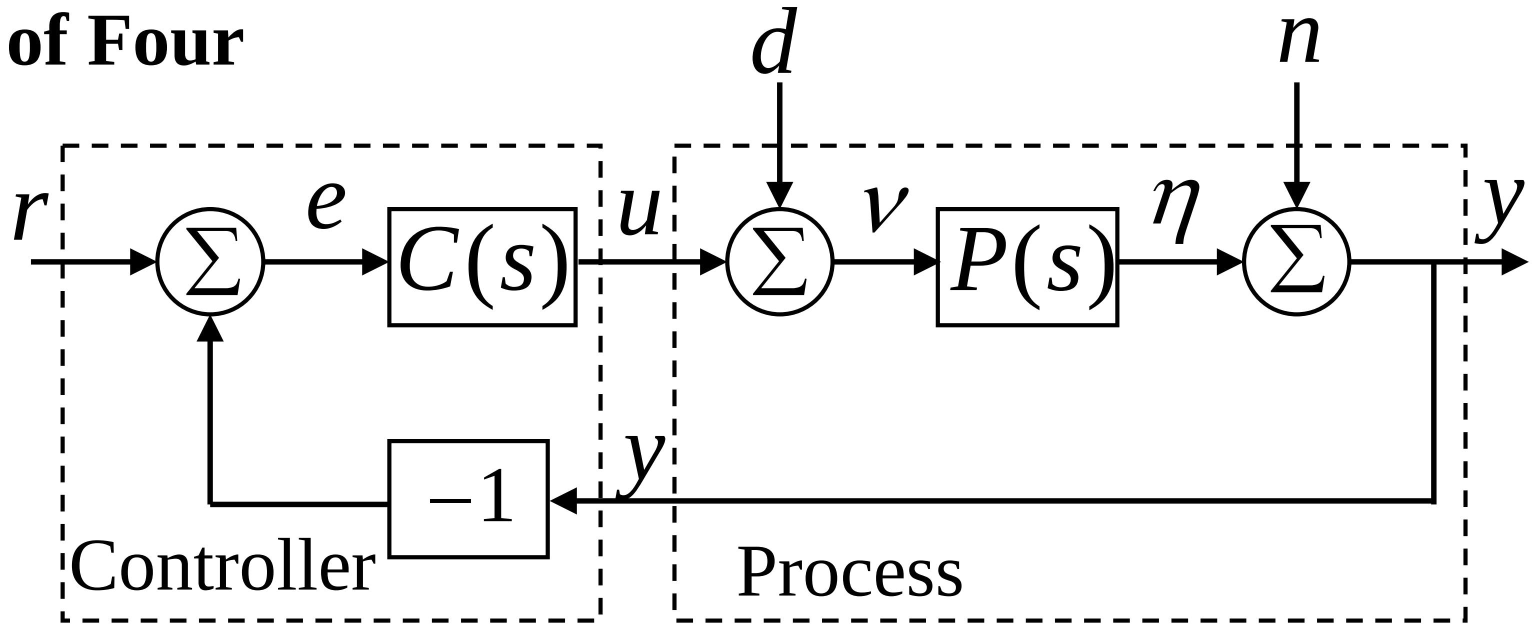


Fig. 11.1

Gang of Four

$$\underset{(d\to v)}{S} = \frac{1}{1+PC} \quad \text{Sensitivity Function}$$

$$\underset{(r\to \eta)}{T} = \frac{PC}{1+PC} \quad \text{Complementary Sensitivity Function}$$

$$\underset{(d\to \eta)}{PS} = \frac{P}{1+PC} \quad \text{Load Sensitivity Function}$$

$$\underset{(n\to u)}{CS} = \frac{C}{1+PC} \quad \text{Noise Sensitivity Function}$$

(11.3)

General Representation*

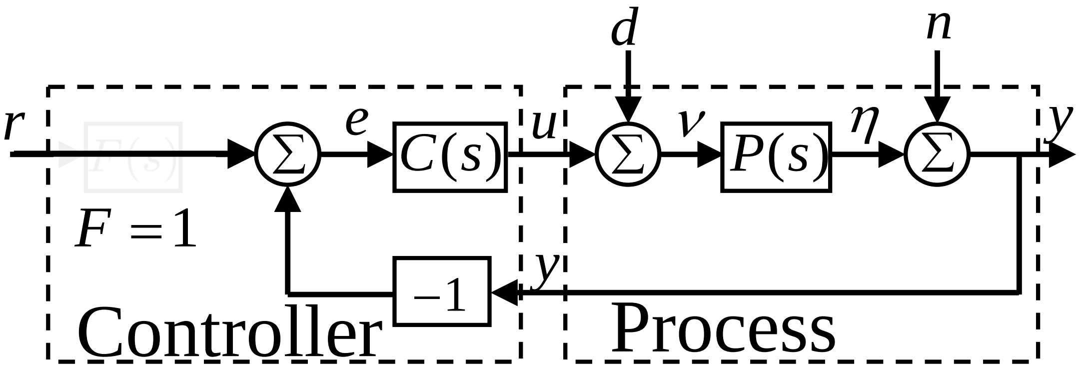


Fig. 11.1

General representation

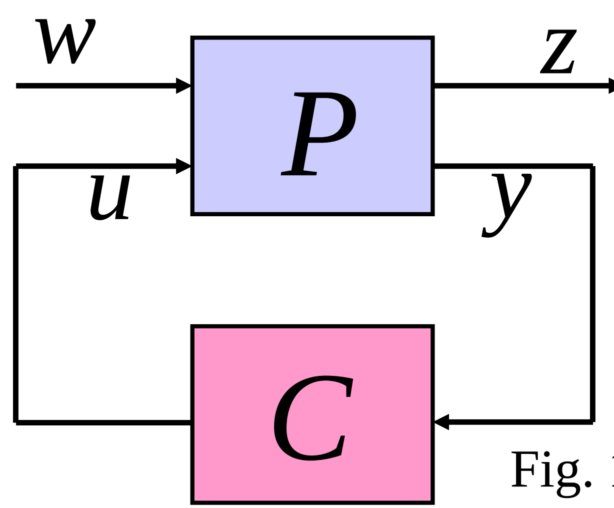


Fig. 11.2

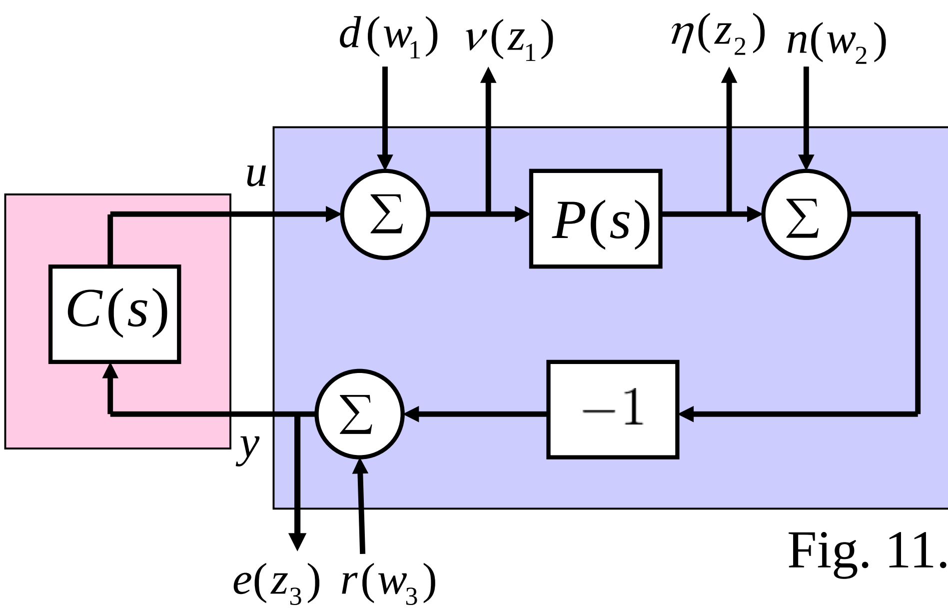


Fig. 11.2

$$P = \begin{bmatrix} 1 & 0 & 0 & 1 \\ P & 0 & 0 & P \\ -P & -1 & 1 & -P \end{bmatrix}$$

$$\mathrm{C} = C \qquad w = \begin{bmatrix} d \\ n \\ r \end{bmatrix} \qquad z = \begin{bmatrix} v \\ \eta \\ e \end{bmatrix}$$

[Ex. 11.1]

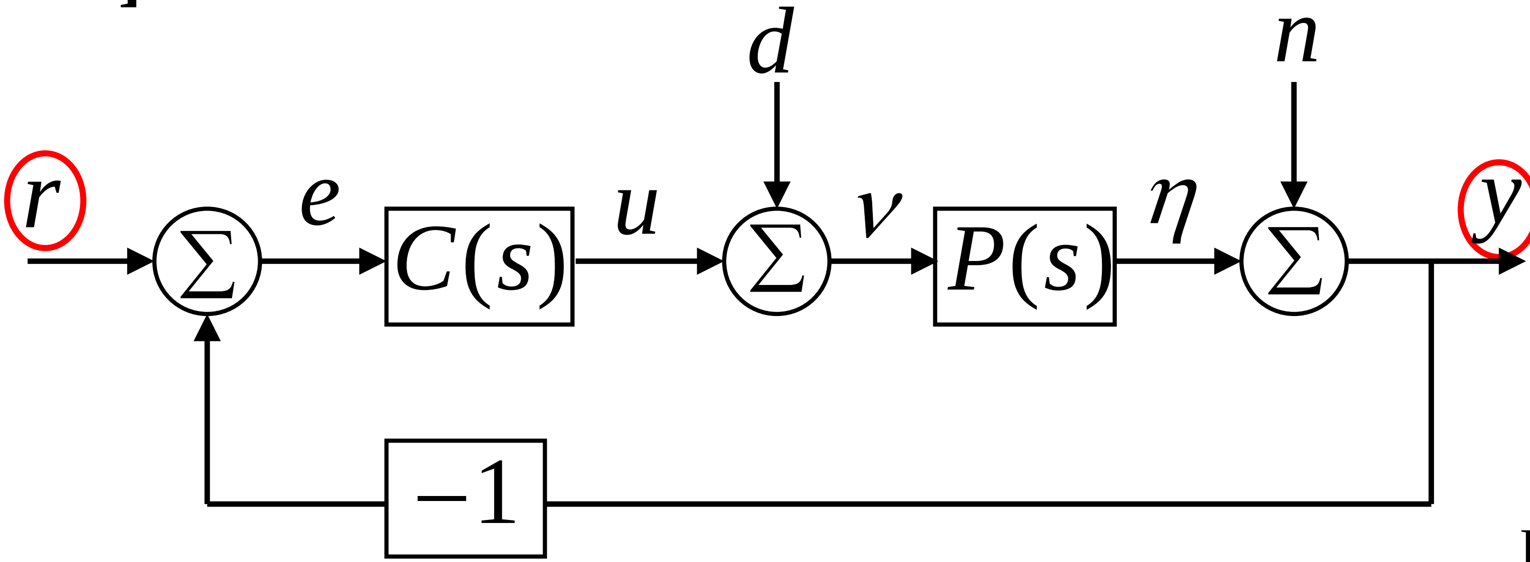


Fig. 11.1

Process

$$P(s) = \frac{1}{s-a}$$

Controller

$$C(s) = \frac{k(s-a)}{s}$$

$$k > 0$$

$$r \rightarrow y$$

$$G_{yr} = ?$$

Stable ?

$$G_{yr}(s) = T(s) = \frac{k}{s+k}, \; k > 0$$

Gang of Four

Sensitivity Function

$$S(s)=\frac{s}{s+k}$$

Load Sensitivity Function

$$PS(s)=\frac{s}{(s+k)(s-a)}$$

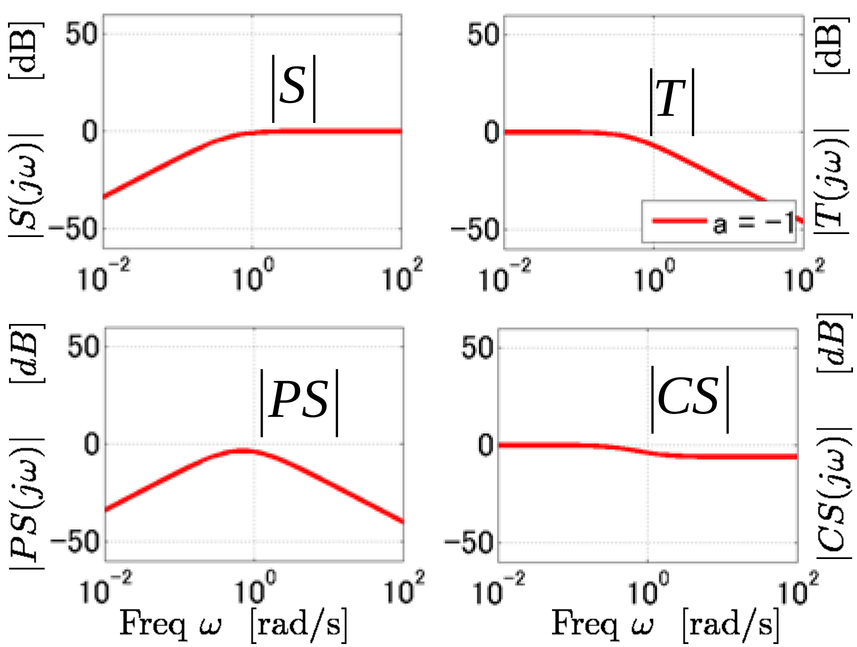
can be unstable

Complementary Sensitivity Function

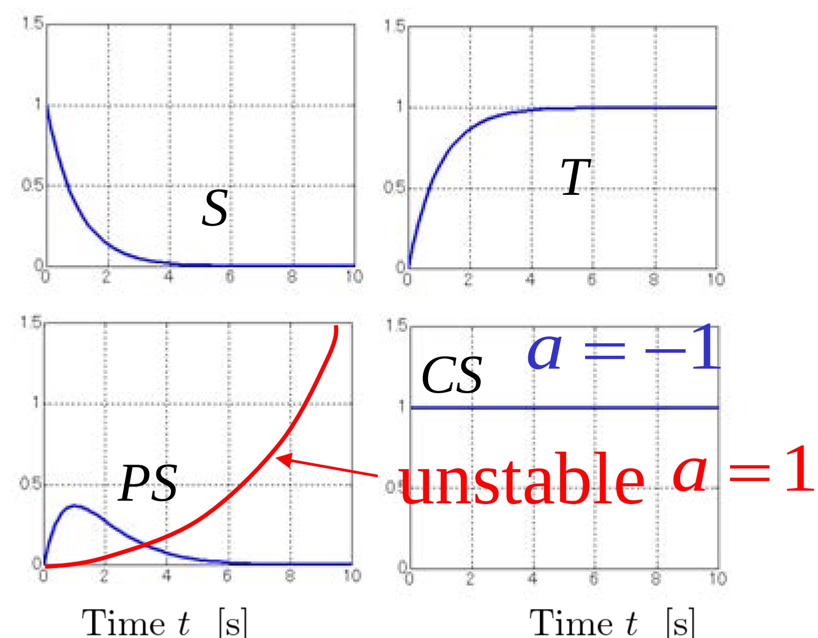
$$T(s)=\frac{k}{s+k}$$

Noise Sensitivity Function

$$CS(s)=\frac{k(s-a)}{s+k}$$



Frequency response



Step response

Internal Stability

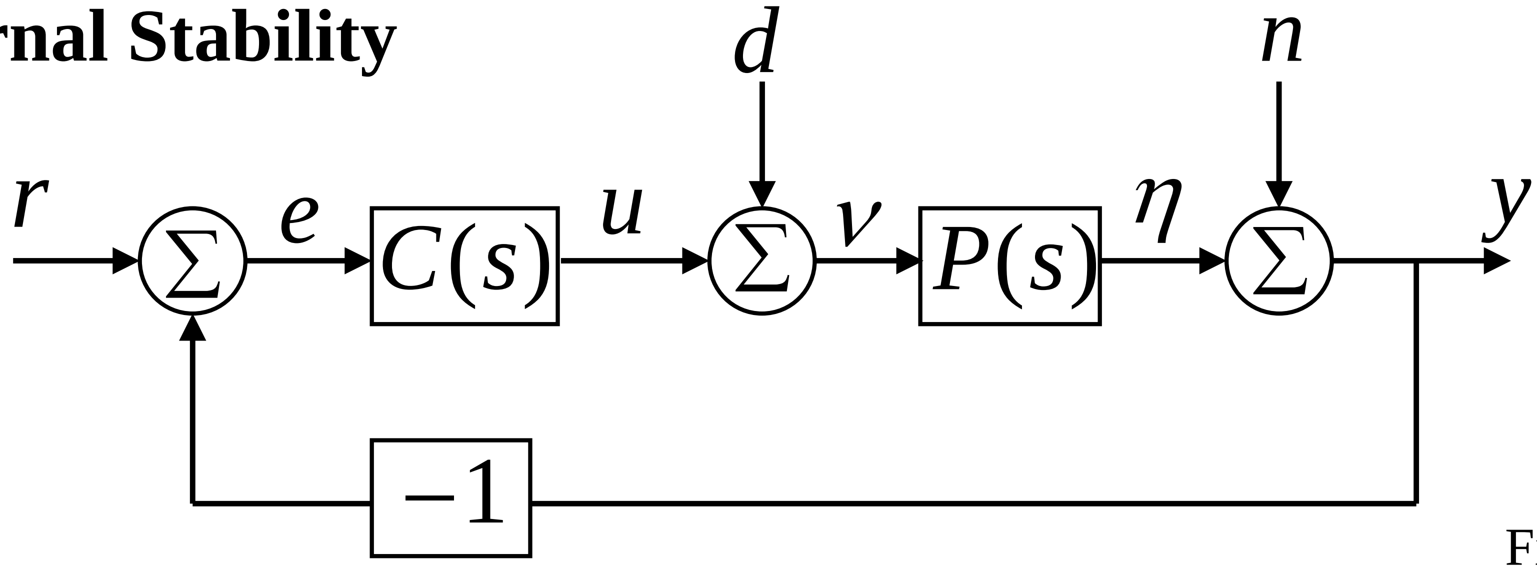


Fig. 11.1

Internal Stability : All of the “Gang of Four” are stable.

$$\underset{(d\to v)}{S} = \frac{1}{1+PC} \quad \text{Sensitivity Function}$$

$$\underset{(r\to \eta)}{T} = \frac{PC}{1+PC} \quad \text{Complementary Sensitivity Function}$$

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(11.3)

Well-posed: $1+P(\infty)C(\infty) \neq 0$

Youla Parameterization (stable process) (§ 12.2)

$$C = \frac{Q}{1 - PQ} \qquad (12.8)$$

"All" stabilizing controllers

$P(s)$: Stable process

$Q(s)$: Stable transfer function (parameter)

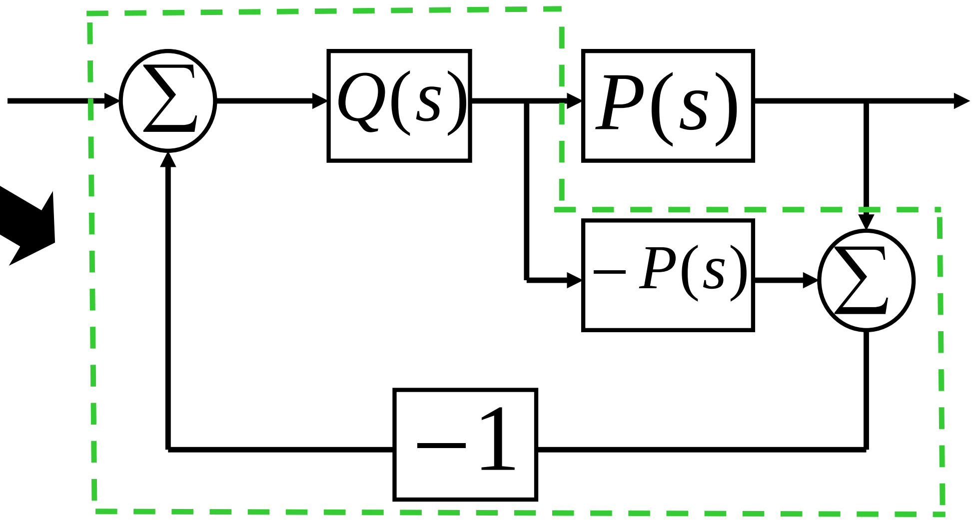


Fig.12.8 (a)

IMC : Internal Model Control

Gang of Four

$$\begin{cases} S = 1 - PQ \\ PS = P(1 - PQ) \\ CS = Q \\ T = PQ \end{cases}$$

All of these 4 transfer functions are stable.

All stabilizing controllers are parameterized by Q

Youla Parameterization (unstable process)

$$P(s)=\frac{A(s)}{B(s)} \quad A(s), B(s): \text{Stable}$$

[Ex.]

$$P(s)=\frac{1}{(s-1)(s-2)} \;\Rightarrow\; A(s)=\frac{1}{(s+1)^2}, B(s)=\frac{(s-1)(s-2)}{(s+1)^2}$$

Coprime Factorization

$$AF_0+BG_0=1 \qquad F_0(s), G_0(s): \text{Stable}$$

[Ex.] $A(s)=\frac{1}{(s+1)^2}, B(s)=\frac{(s-1)(s-2)}{(s+1)^2}$

$$\Rightarrow\; F_0(s)=\frac{19s-11}{s+1}, G_0(s)=\frac{s+6}{s+1}$$

Youla Parameterization (unstable process)

$$C = \frac{F_0 + QB}{G_0 - QA} \quad (12.9)$$

: "All" stabilizing controllers

$Q(s)$: Stable

[Ex.] $P(s) = \dfrac{1}{(s-1)(s-2)}$ ⇒ $C(s) = \dfrac{19s - 11}{s + 6} \quad (Q = 0)$

Exercise: Internal Stability

Gang of Four

$$S = \frac{B(G_0 - QA)}{AF_0 + BG_0} \qquad T = \frac{A(F_0 + QB)}{AF_0 + BG_0}$$

$$PS = \frac{A(G_0 - QA)}{AF_0 + BG_0} \qquad CS = \frac{B(F_0 + QB)}{AF_0 + BG_0}$$

1st Lecture

11 Frequency Domain Design

11.1 Sensitivity Functions (pp.315 to 319)

(12.2 Youla Parameterization) (pp.352 to 358)

Keyword : Basic Feedback Loop, Gang of Four
Internal Stability, Youla Parameterization

11.3 Performance Specifications (pp.322 to 326)

(12.3 Performance in the Presence of Uncertainty) (pp.358 to 361)

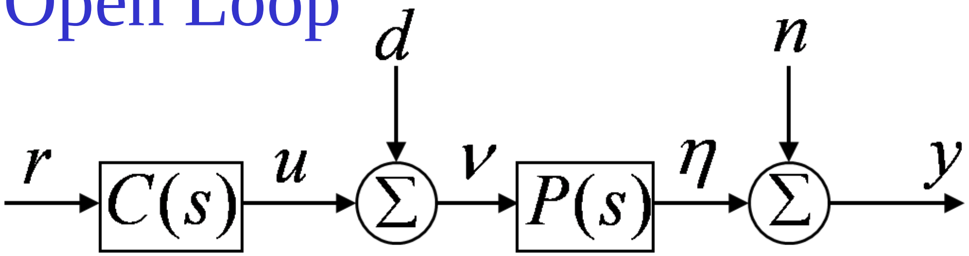
(11.5 Fundamental Limitation) (pp.331 to 340)

Keyword : Sensitivity Function

11.3 Performance Specifications

Disturbance Attenuation ~ Open Loop vs. Closed Loop ~

Open Loop



$$d \to y$$

$$G_{yd} = P$$

Closed Loop
(feedback)

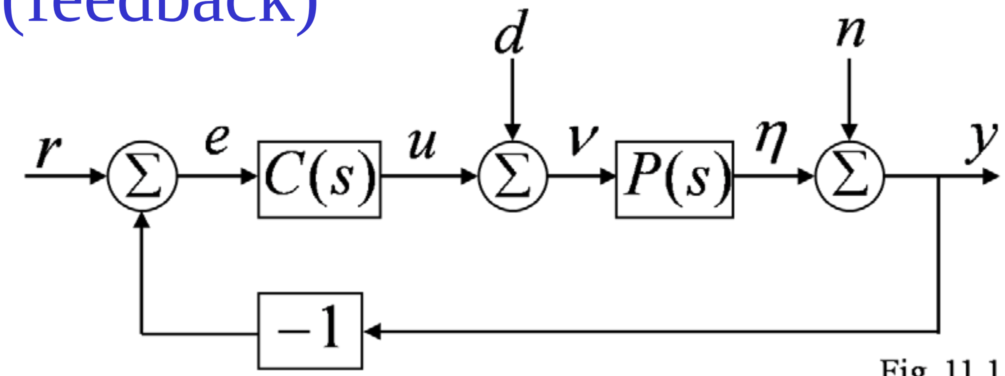


Fig. 11.1

$$d \to y$$

$$G_{yd} \overset{?}{=} \frac{P}{1+PC}$$

$$= PS$$

The sensitivity function $S(j\omega)$ tells how the variations in the outputs are influenced by feedback

$$|S(j\omega)| < 1 \quad \longleftrightarrow \quad \text{Disturbances are attenuated}$$

Process Variations (§ 12.3)

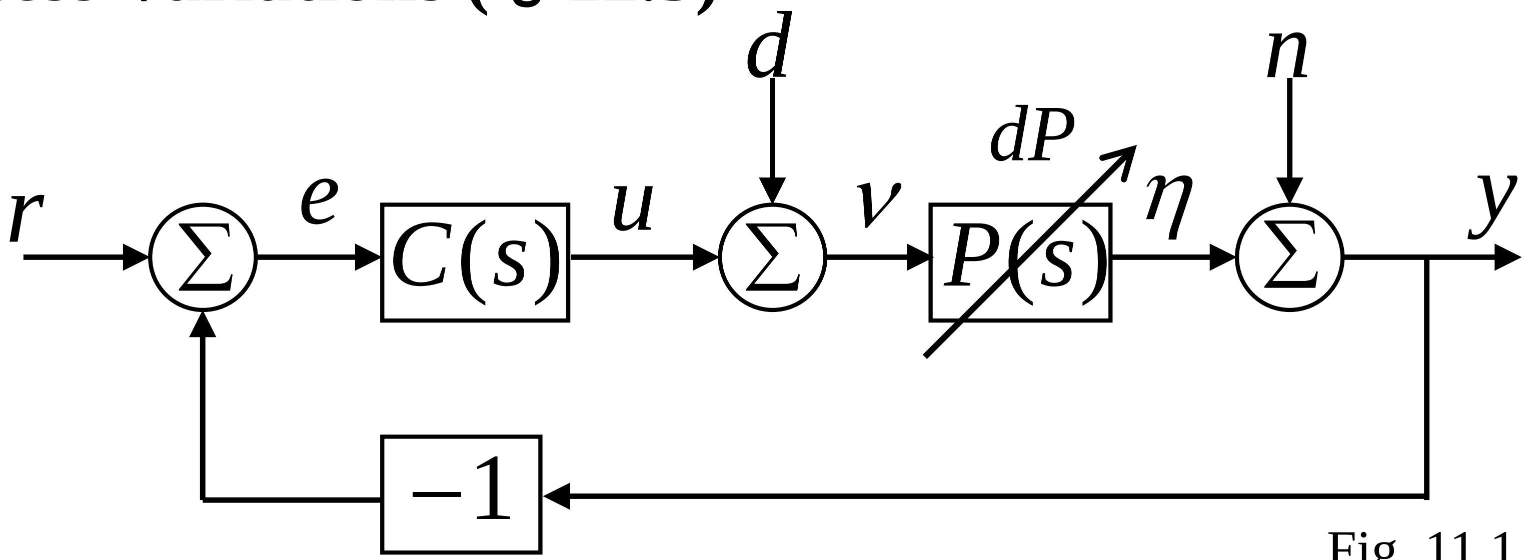


Fig. 11.1

$d \rightarrow y$

$$G_{yd} = \frac{P}{1+PC} = PS \quad \Longrightarrow \quad \frac{dG_{yd}}{G_{yd}} = S\frac{dP}{P} \qquad (12.11)$$

Exercise

The response to load disturbances is insensitive to process variations for frequencies where $|S(j\omega)|$ is small

Process Variations (§ 12.3)

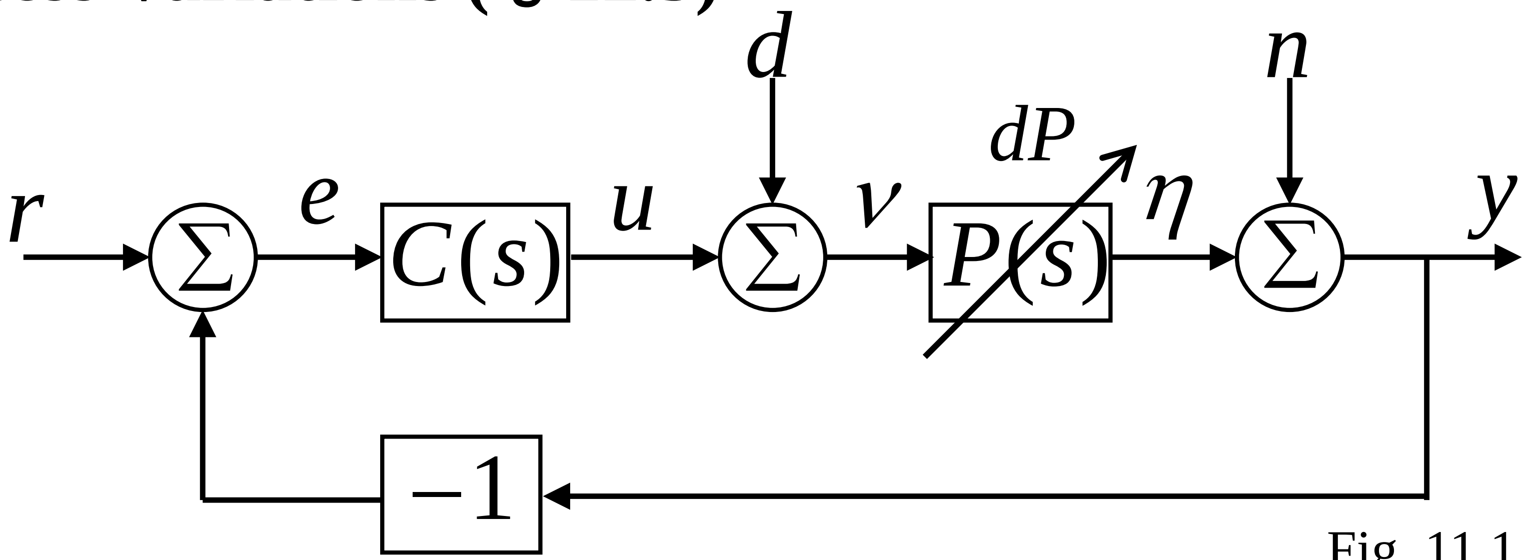


Fig. 11.1

$r \rightarrow y$

$$G_{yr} = \frac{PC}{1+PC} = T \quad \Rightarrow \quad \frac{dG_{yr}}{G_{yr}} = S\,\frac{dP}{P} \qquad (12.15)$$

Exercise

Benefits of Feedback

- Disturbance Attenuation
- Insensitivity to Plant Variations
- Stabilization

Waterbed Effect (§ 11.5)

If the loop transfer function has no right half-plane poles…

$$\int_0^\infty \log|S(j\omega)|d\omega = 0$$

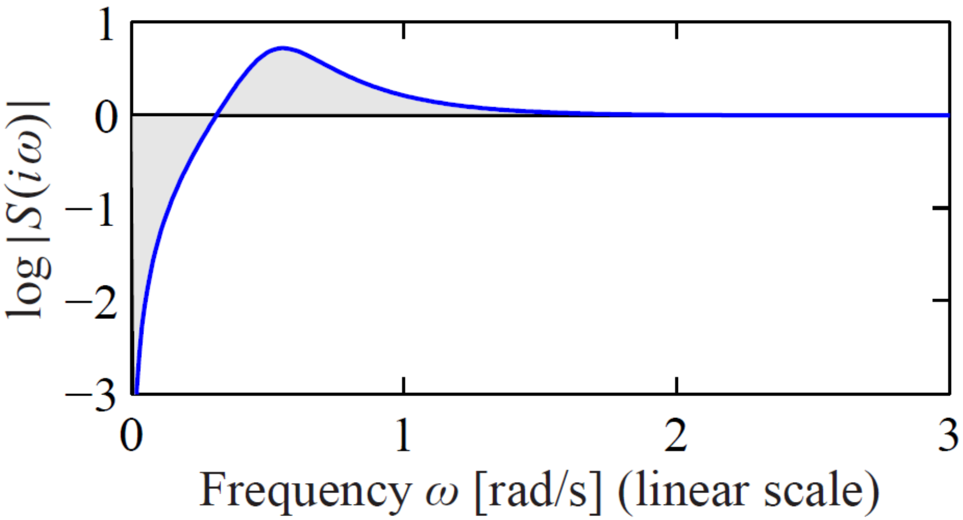
… **Conservation Law**

$\log|S| > 0 \leftrightarrow |S| > 1$

$\log|S| < 0 \leftrightarrow |S| < 1$

There exists a range of freq. such that $|S| > 1$

… **Waterbed Effect**



Bode's Integral Formula (§ 11.5)

Theorem 11.1 Bode's Integral Formula

Assume that the loop transfer function $L(s)$ of a feedback system goes to zero faster than $1/s$ as $s \to \infty$, and let $S(s)$ be the sensitivity function. If the loop transfer function has poles p_k in the right half-plane, then the sensitivity function satisfies the following integral:

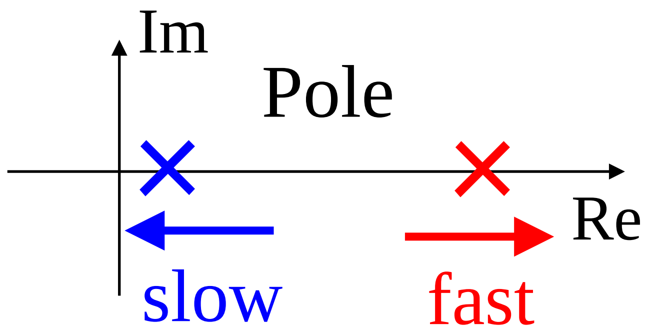
$$\int_0^\infty \log|S(j\omega)|d\omega = \int_0^\infty \log\frac{1}{|1+L(j\omega)|}d\omega = \pi\sum p_k \qquad (11.19)$$

Fundamental Limitation

RHP poles

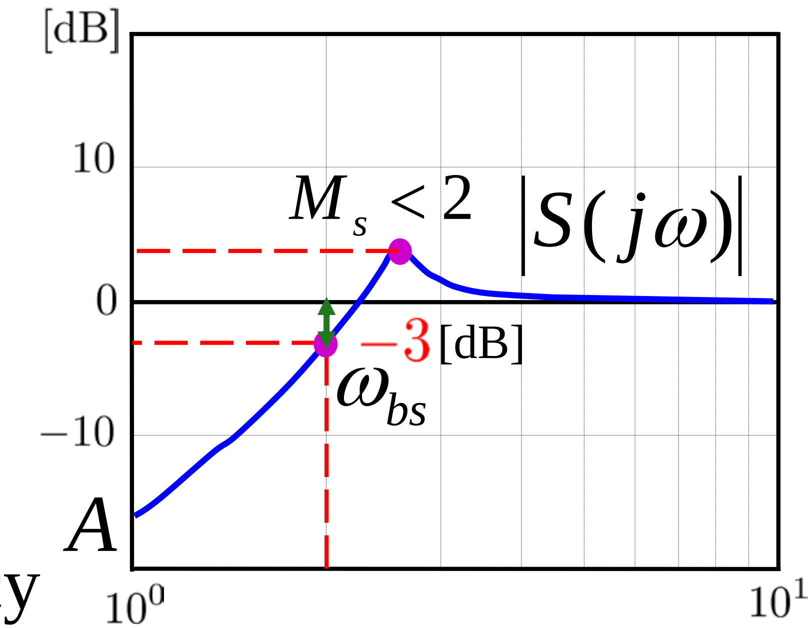
fast (big): worse

slow (small): better



Benefits of Feedback

- Disturbance Attenuation
- Insensitivity to Plant Variations



Sensitivity function $S(j\omega)$

ω_{bs} : Sensitivity Bandwidth Frequency

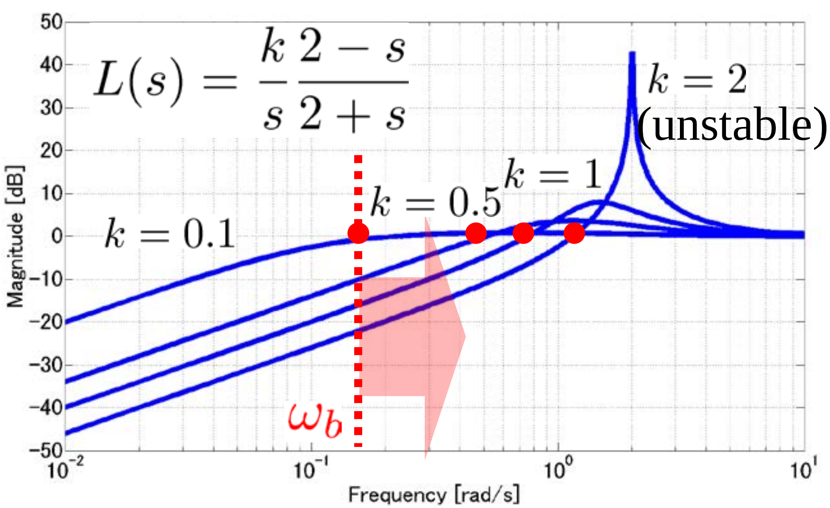
$$|S(j\omega)| = \frac{1}{\sqrt{2}} \quad (-3[\text{dB}])$$

M_s: Maximum Peak Magnitude of $S(j\omega)$

$$M_s = \max_{\omega} |S(j\omega)| \qquad M_s < 2$$

A : Maximum Steady-state Tracking Error Type 1: $A = 0$

[Ex.]



Sensitivity function $S(j\omega)$

$$|S(j\omega)| < 1 \left(S = \frac{1}{1+L} \right) \leftrightarrow |1 + L(j\omega)| > 1$$

s_m : the shortest distance from the Nyquist curve to the critical point (-1)

$M_s = 1/s_m$: maximum sensitivity

s_m : stability margin

$M_s < 2$

$0.5 < s_m < 0.8$

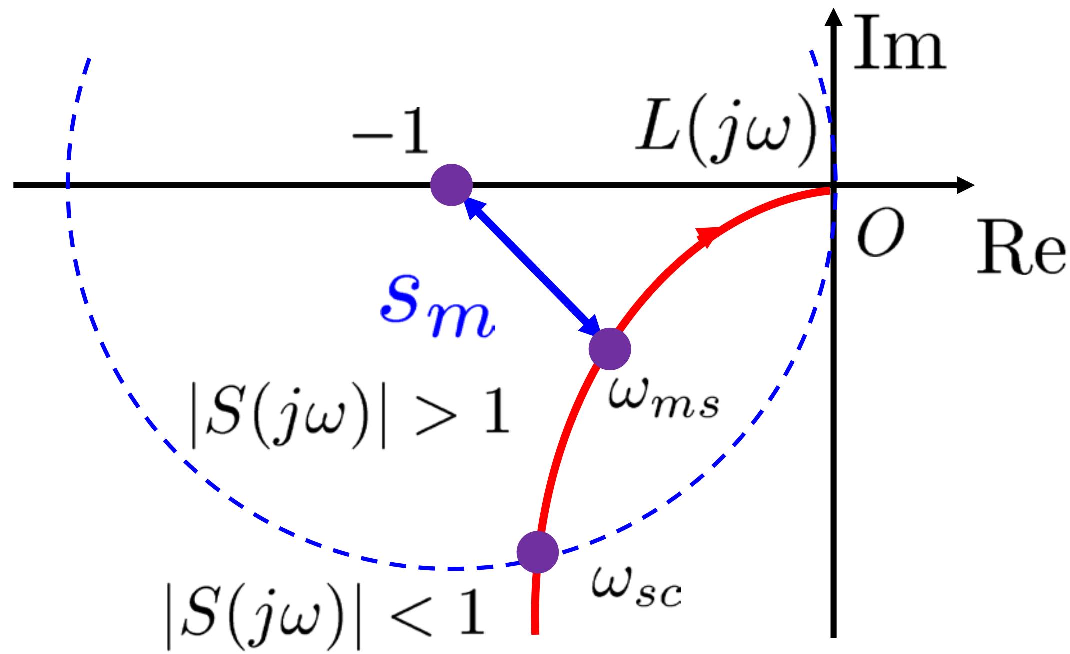


Fig. 11.6(b)

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