Analysis and Design of Linear Control System –Part2-

Instructor: Prof. Masayuki Fujita

2nd Lecture

12 Robust Performance

12.1 Modeling Uncertainty (pp.347 to 352)

(9.2 The Nyquist Criterion) (pp.270 to 278)

(9.3 Stability Margins) (pp.278 to 282)

12.2 Stability in the Presence of Uncertainty (pp.352 to 358)

Keyword : Modeling Uncertainty
Robust Stability

12.2 Stability in the Presence of Uncertainty

(12.3 Performance in the Presence of Uncertainty) (pp.358 to 361)

(11.5 Fundamental Limitation) (pp.331 to 340)

Keyword : Complementary Sensitivity Function
Small Gain Theorem

12.1 Modeling Uncertainty

- **Parametric uncertainty**

  parameters describing the system are unknown

  Mass of a car changes with the number of passengers

  mass $1600 < m < 2000$

  (3rd) (4th) (5th)

  gear ratio $\alpha = 10, 12, 14$ speed $10 \leq v_e \leq 40$

$$\theta = 3^{\circ}$$

Disturbance response

Fig. 3.3

Modeling Uncertainty

- **Parametric uncertainty**

  parameters describing the system are unknown

  → The design based on a simple nominal model will give satisfactory control.

  Ex. 12.1

- **Unmodeled dynamics**

  neglected mechanisms such that the simple model does not include.

  - detailed model of the engine dynamics
  - slight delays that can occur in electronically controlled engines

Unmodeled Dynamics

Vibration mode

$$\tilde{P} = \underbrace{\frac{0.5}{s}}_{P} + \underbrace{\sum_{i=1}^{4} \frac{0.2s}{s^2 + 2\zeta_i \omega_i s + \omega_i^2}}_{\Delta}$$

vibration mode

http://mobile.jaxa.jp/gallery_list/index.php?category=iss

$\omega_1 = 0.2, \omega_2 = 0.5, \omega_3 = 2$ → **Unmodeled dynamics**

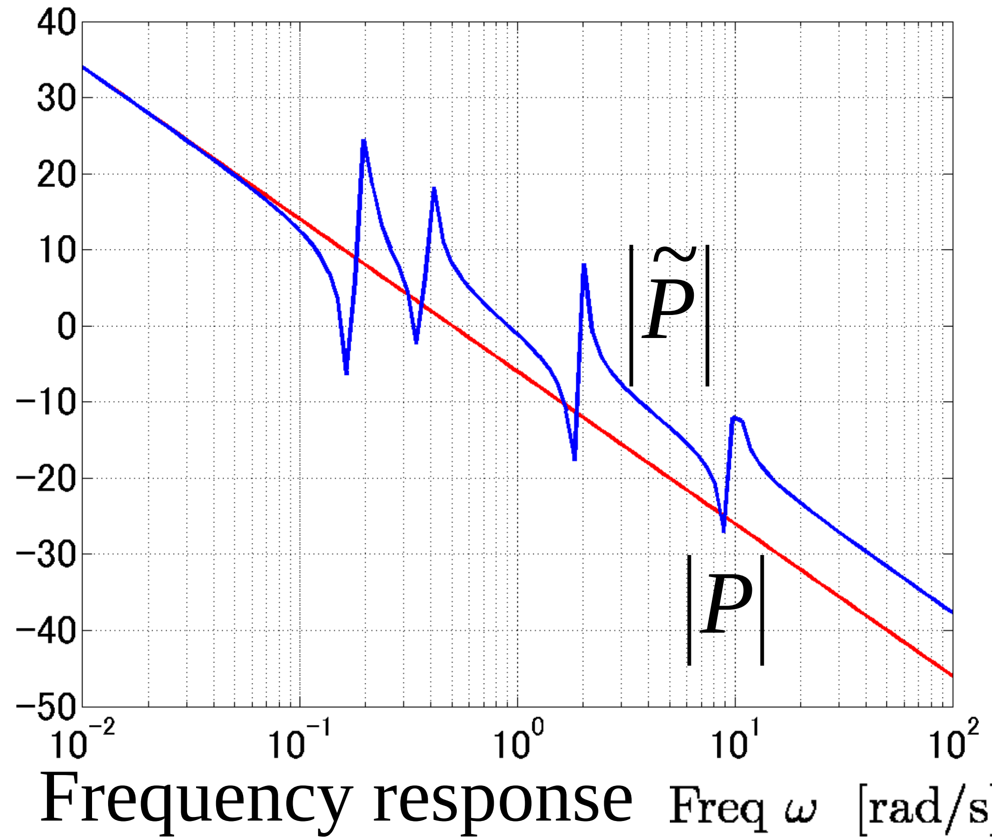
$\omega_4 = 10, \zeta_i = 0.02$

Nominal System P

(Rigid body mode)

$$P = \frac{0.5}{s}$$

$$\tilde{P} = P + \Delta$$

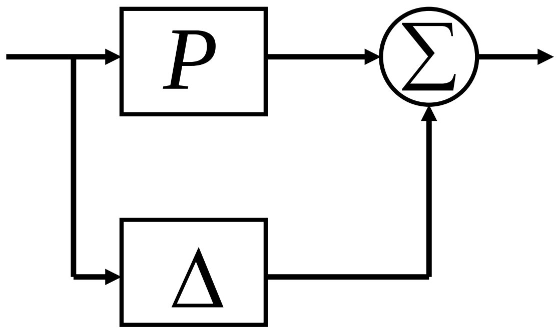


Frequency response

Unmodeled Dynamics

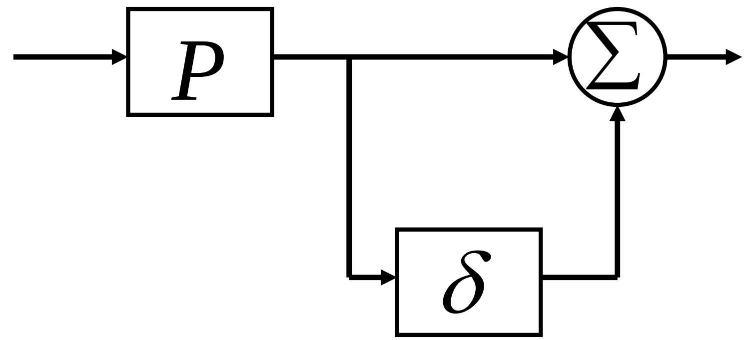
Additive perturbations

$$\tilde{P} = P + \Delta$$



Multiplicative perturbations

$$\tilde{P} = (1+\delta)P$$

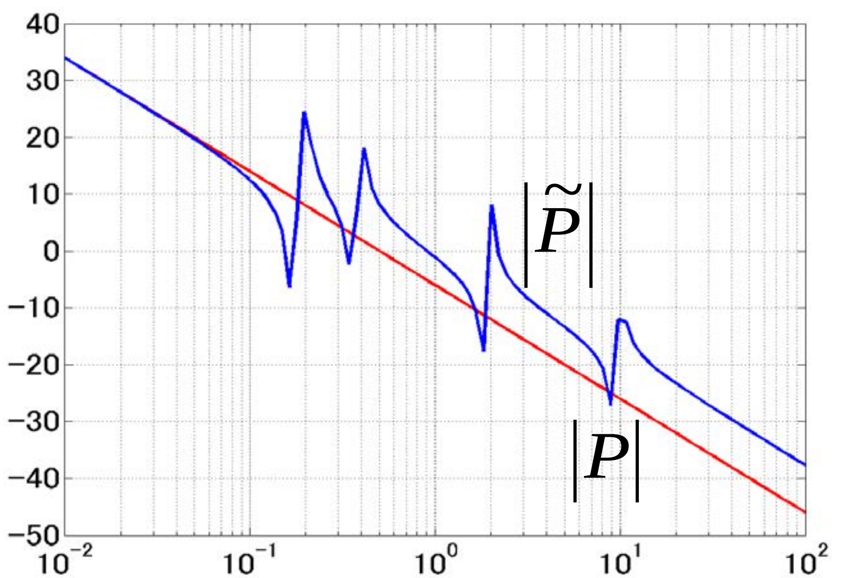


$$\delta := \Delta / P$$

P : nominal model

$\tilde{P}$: actual model

Δ, δ : unmodeled dynamics



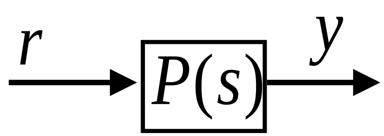
Frequency response

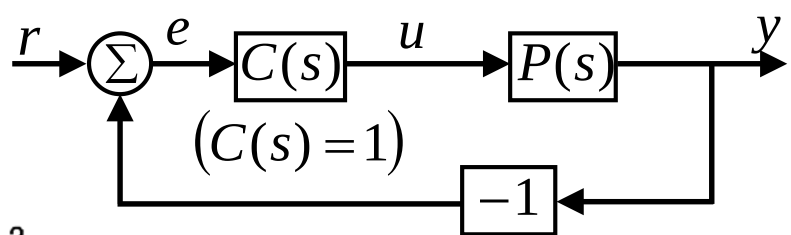
Freq ω [rad/s]

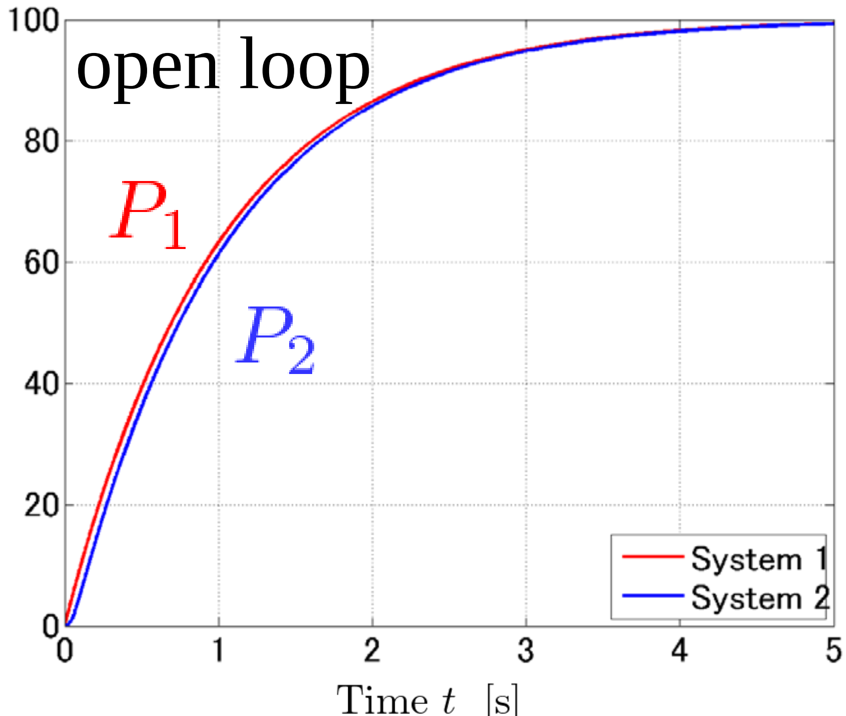
[When Are Two Systems Similar ?]

[Ex. 12.2] Similar in Open Loop but Large Differences in Closed Loop

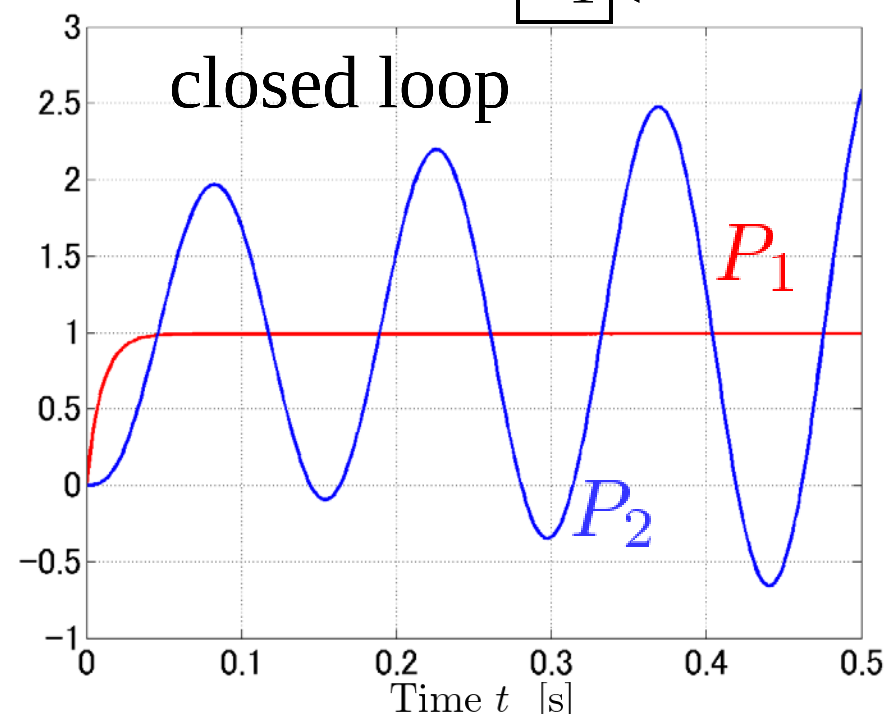
$$P_1(s) = \frac{k}{s+1} \qquad P_2(s) = \frac{k}{(s+1)(sT+1)^2} \qquad \begin{aligned} T &= 0.025 \\ k &= 100 \end{aligned} \tag{12.1}$$







(a) Step response (open loop)



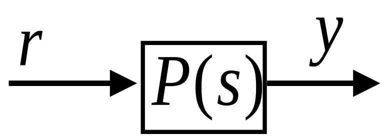
(b) Step response (closed loop)

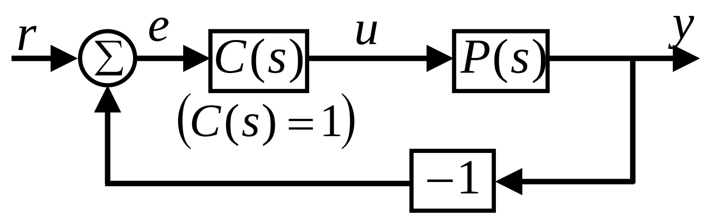
Fig. 12.3(a)

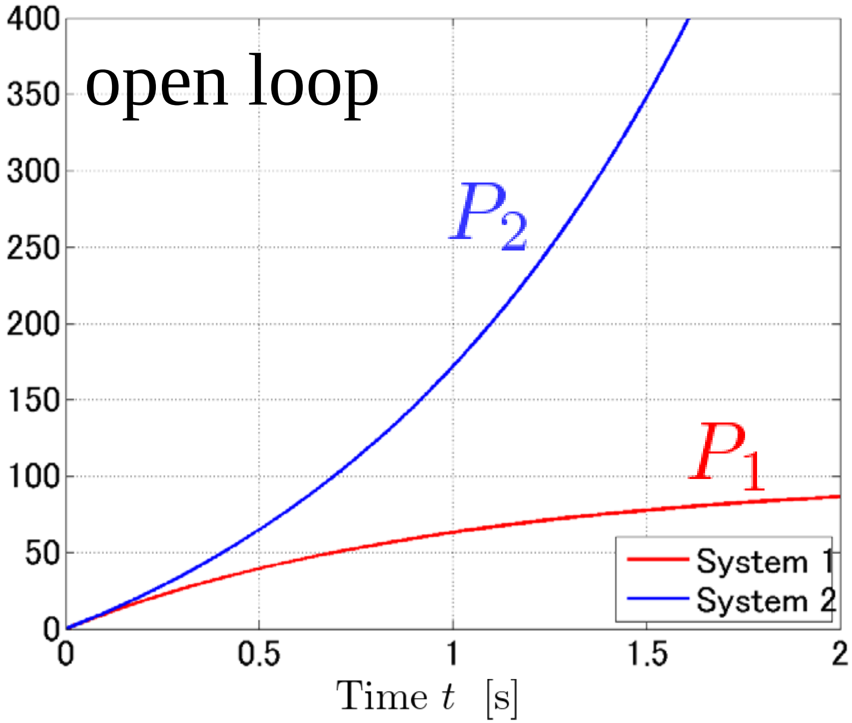
[When Are Two Systems Similar ?]

[Ex. 12.3] Different in Open Loop but Similar in Closed Loop

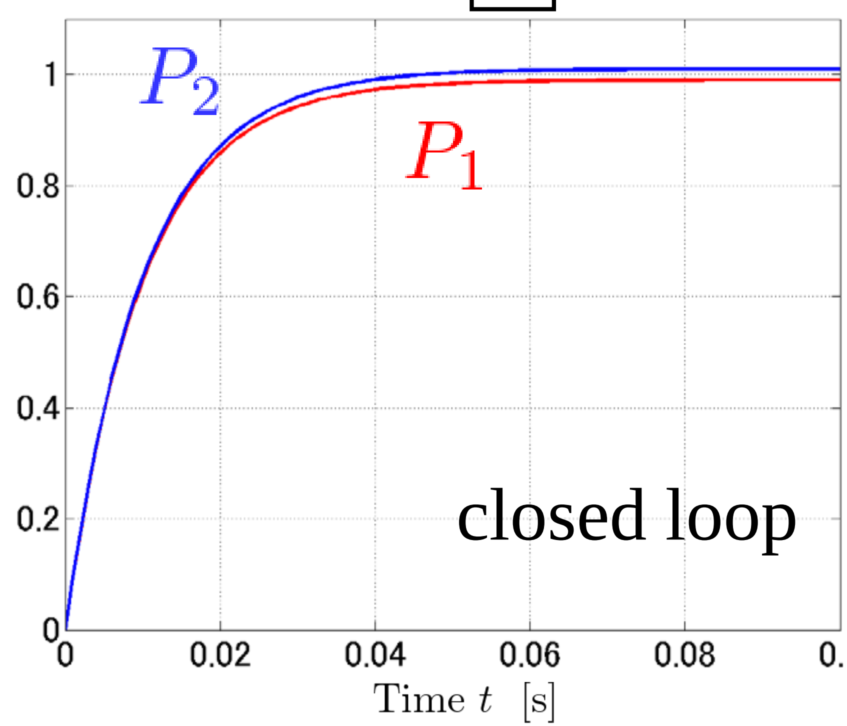
$$P_1(s) = \frac{k}{s+1} \qquad P_2(s) = \frac{k}{(s-1)} \qquad k = 100 \tag{12.2}$$







(a) Step response (open loop)



(b) Step response (closed loop)

Fig. 12.3(b)

Nyquist Criterion (§ 9.2)

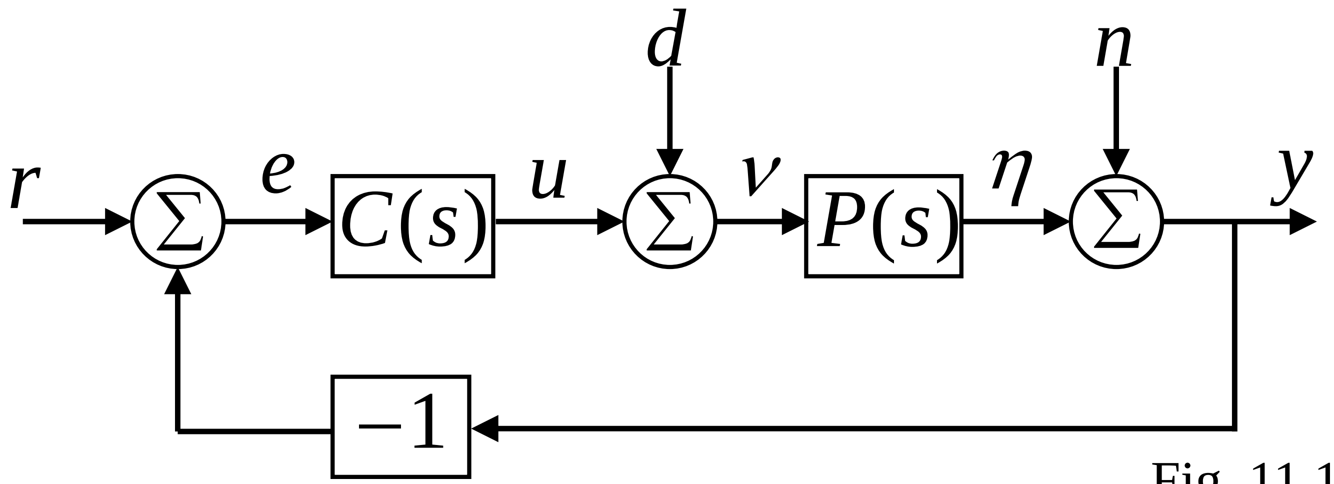
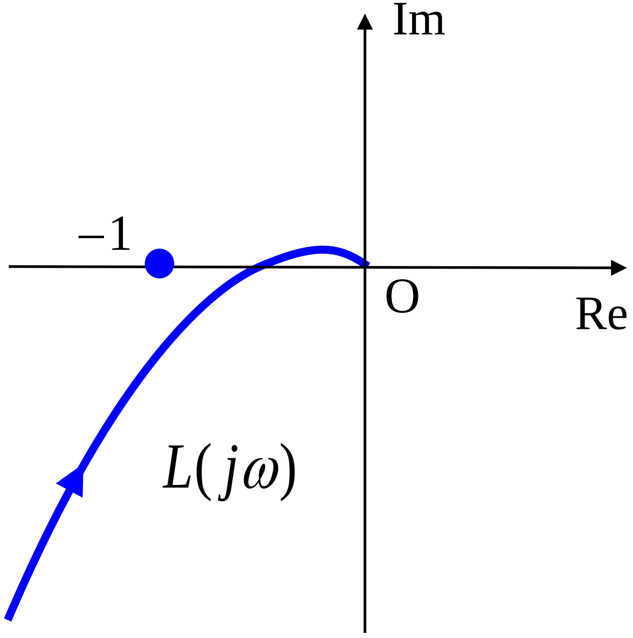


Fig. 11.1

Loop Transfer Function

$$L = PC$$



Nyquist's Stability Theorem

Theorem 9.1 and 9.2

Stability Margin (§ 9.3)

Gain Margin

$$g_m = 1/\left|L(i\omega_{pc})\right| \quad (9.5) \qquad g_m = 2-5$$

(6-14 dB)

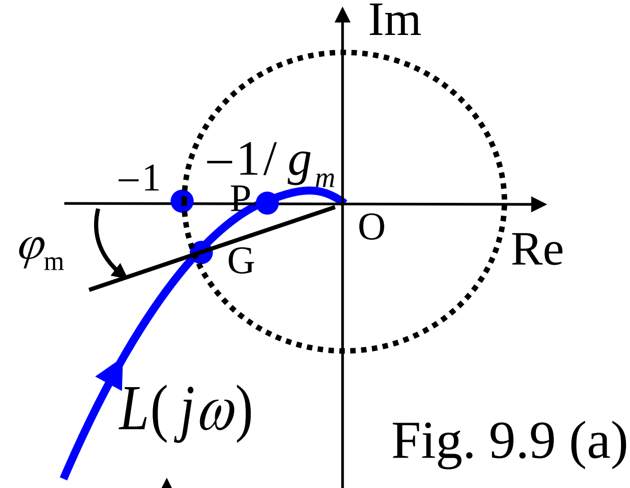


Fig. 9.9 (a)

Phase Margin

$$\varphi_m = \pi + \arg L(i\omega_{gc}) \quad (9.6) \qquad \varphi_m = 30^\circ - 60^\circ$$

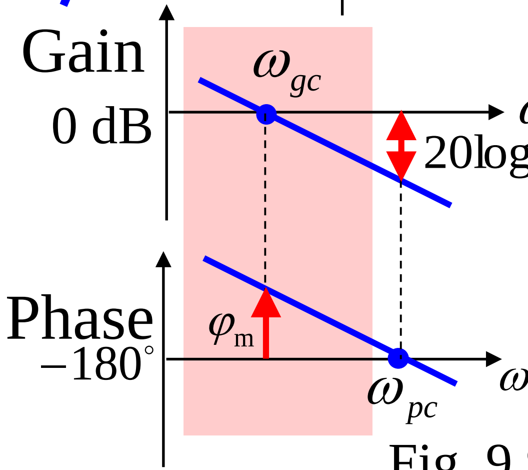


Fig. 9.9 (b)

Stability Margin s_m

$$s_m = 1/M_s \qquad 0.5 < s_m < 0.8$$

$$(M_s = 1/s_m : \text{maximum sensitivity})$$

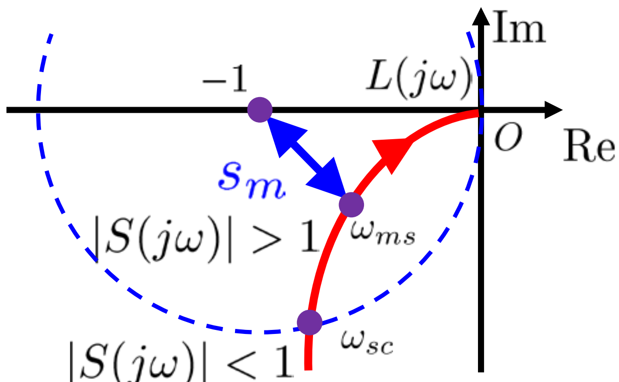
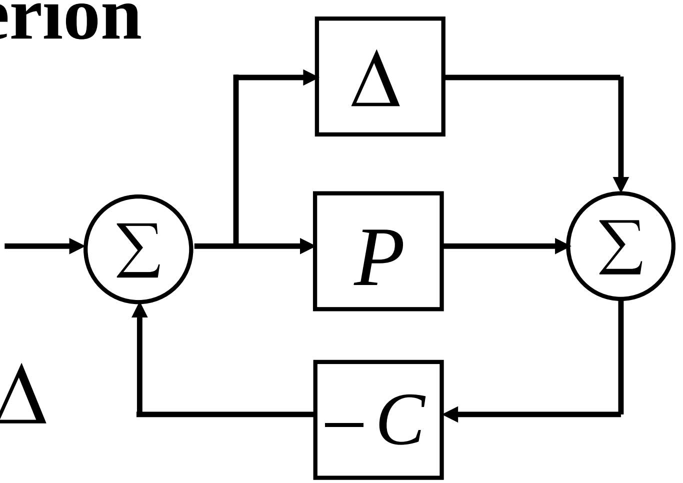


Fig. 11.6(b)

Robust Stability Using Nyquist's Criterion

Additive Uncertainty

$$P \longrightarrow P+\Delta \qquad \Delta : \text{stable perturbations}$$

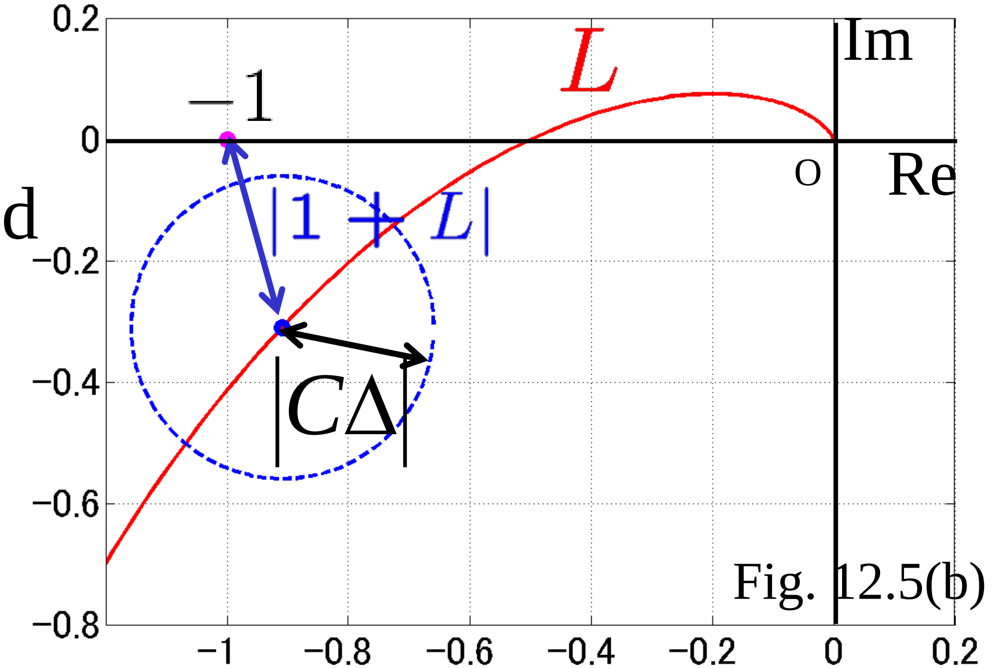


Loop Transfer Function

$$L = PC \longrightarrow (P+\Delta)C = PC + C\Delta$$
$$= L + C\Delta$$

Perturbed Nyquist curve doesn't reach -1 (i.e. perturbed closed loop is stable) when

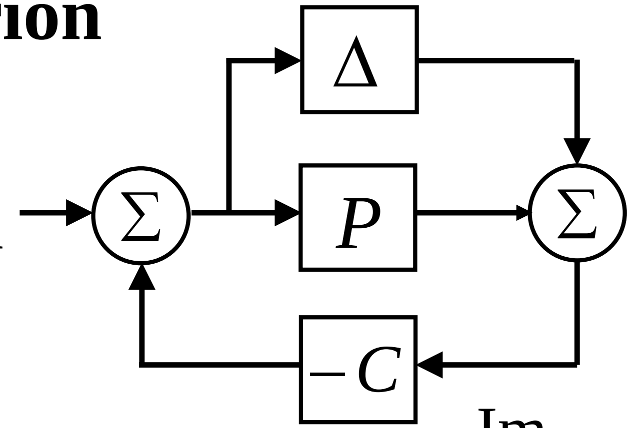
$$|C\Delta| < |1+L|$$



Nyquist plot (additive uncertainty)

Robust Stability Using Nyquist's Criterion

Additive Uncertainty



Perturbed Nyquist curve doesn't reach -1 when

$$|C\Delta| < |1+L| \qquad |\Delta| < \left|\frac{1+PC}{C}\right|$$

$$|\Delta| < \frac{1}{|CS|} \quad \left(\because\ S = \frac{1}{1+PC}\right)$$

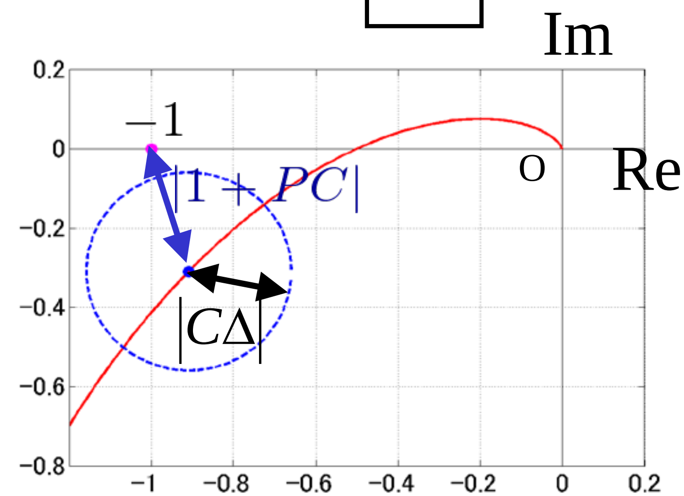
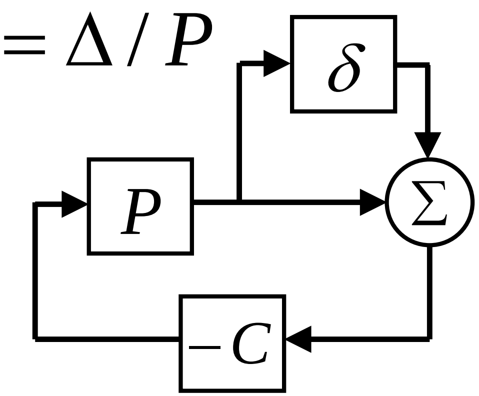


Fig. 12.5(b)

Multiplicative Uncertainty

$$\delta := \Delta / P$$



$$|\delta| = \left|\frac{\Delta}{P}\right| < \left|\frac{1+PC}{PC}\right| = \frac{1}{|T|} \quad \left(\because\ T = \frac{PC}{1+PC}\right)$$

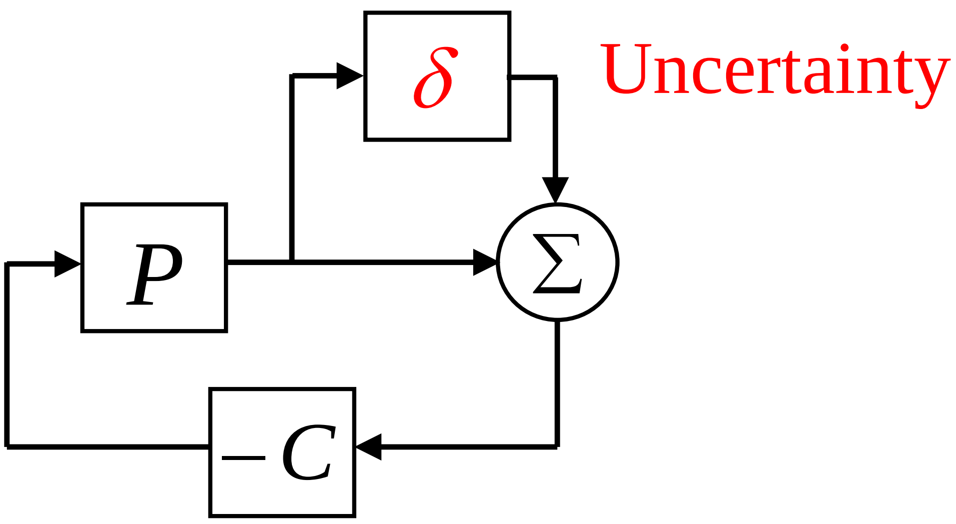
Robust Stability Using Nyquist's Criterion

Sufficient condition for robust stability

$$\left|\delta(j\omega)\right| < \frac{1}{\left|T(j\omega)\right|} \quad \forall \omega \geq 0 \qquad (12.6)$$

small $\left|T(j\omega)\right| \longrightarrow$ big $\left|\delta(j\omega)\right|$ is allowed

big $\left|T(j\omega)\right| \longrightarrow$ small $\left|\delta(j\omega)\right|$ is allowed



Performance in the Presence of Uncertainty (§ 12.3)

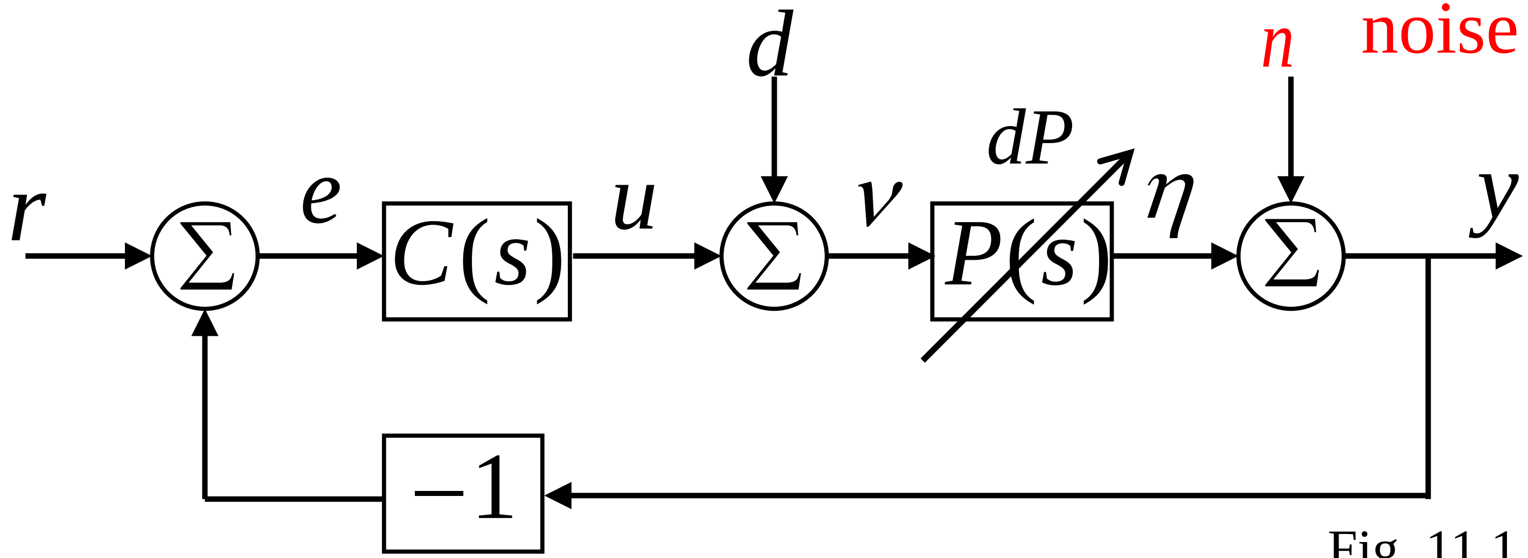


Fig. 11.1

$n \to u$

$$G_{un} = -\frac{C}{1+PC} = -\frac{T}{P} \quad \Rightarrow \quad \frac{dG_{un}}{G_{un}} = -T\frac{dP}{P} \qquad (12.13)$$

Exercise

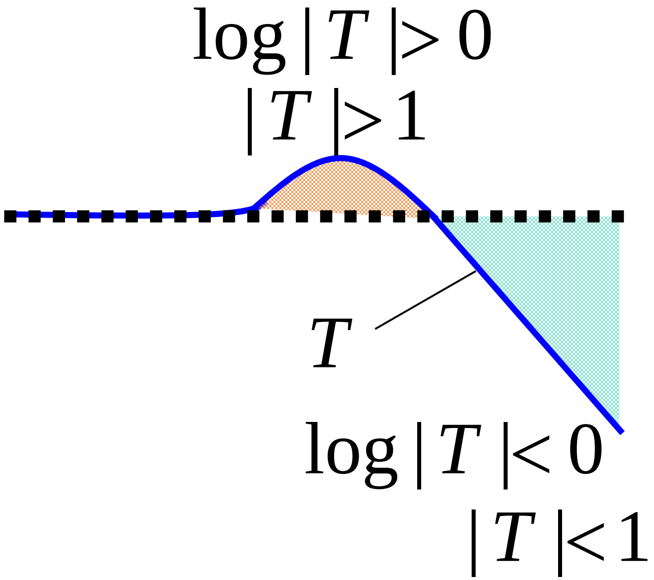
Measurement noise typically has high frequencies

Bode's Integral Formula (§ 11.5)

Complementary sensitivity function:

$$\int_0^{\infty} \frac{\log\left|T(j\omega)\right|}{\omega^2} d\omega = \pi \sum \frac{1}{z_i} \qquad (11.20)$$

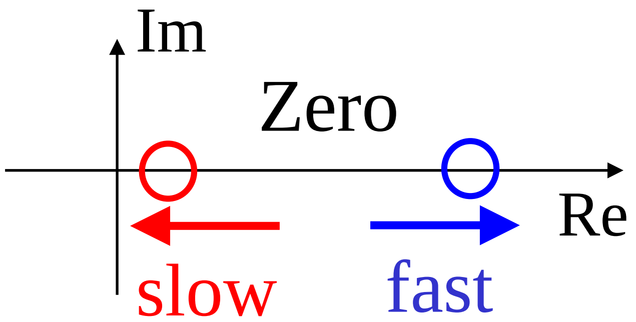
where the summation is over all right half-plane zeros.



RHP zeros

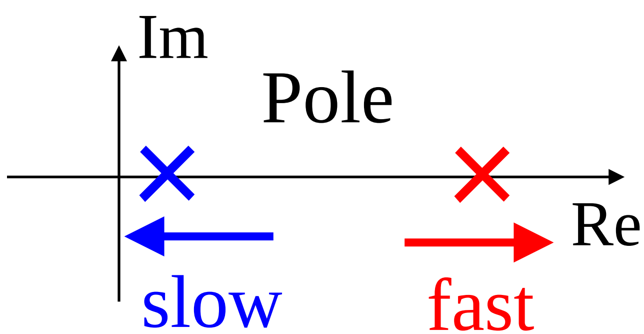
fast (big): better

slow (small): worse



Sensitivity function: (1st lecture)

$$\int_0^{\infty} \log\left|S(j\omega)\right| d\omega = \pi \sum p_k \qquad (11.19)$$



Complementary Sensitivity Function

$$T(s) = \frac{P(s)C(s)}{1+P(s)C(s)} \qquad \left(S+T = \frac{1}{1+PC} + \frac{PC}{1+PC} = 1 \right)$$

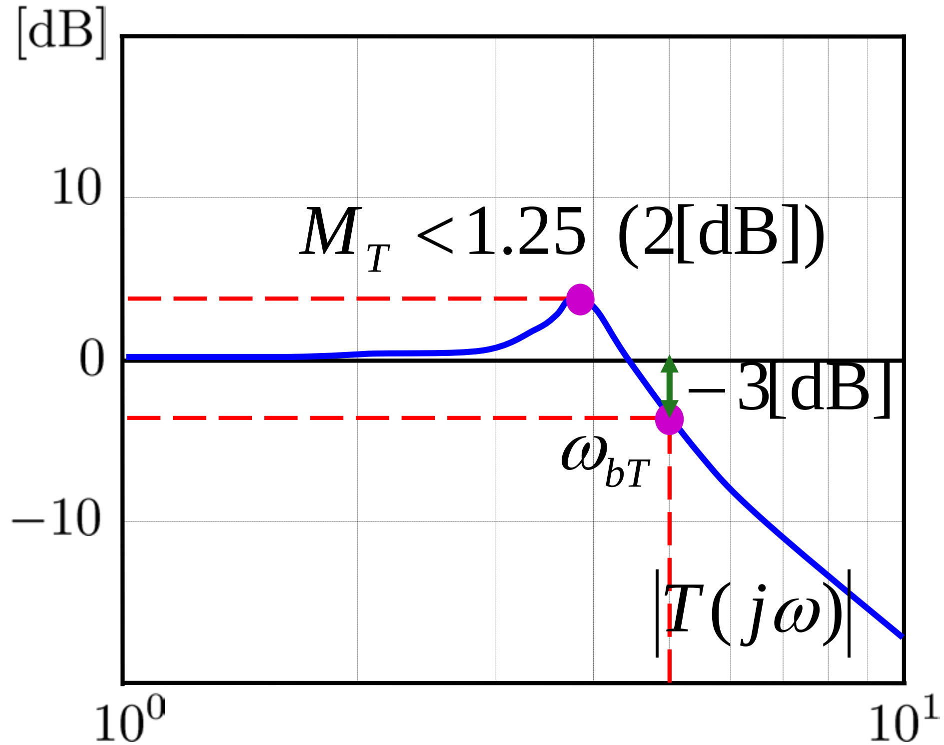
ω_{bT} : Complementary Sensitivity Bandwidth Frequency

$$|T(j\omega)| = \frac{1}{\sqrt{2}} \quad (-3[\mathrm{dB}])$$

M_T : Maximum Peak Magnitude of $T(j\omega)$

$$M_T = \max_{\omega} |T(j\omega)|$$

$$M_T < 1.25 \quad (2[\mathrm{dB}])$$



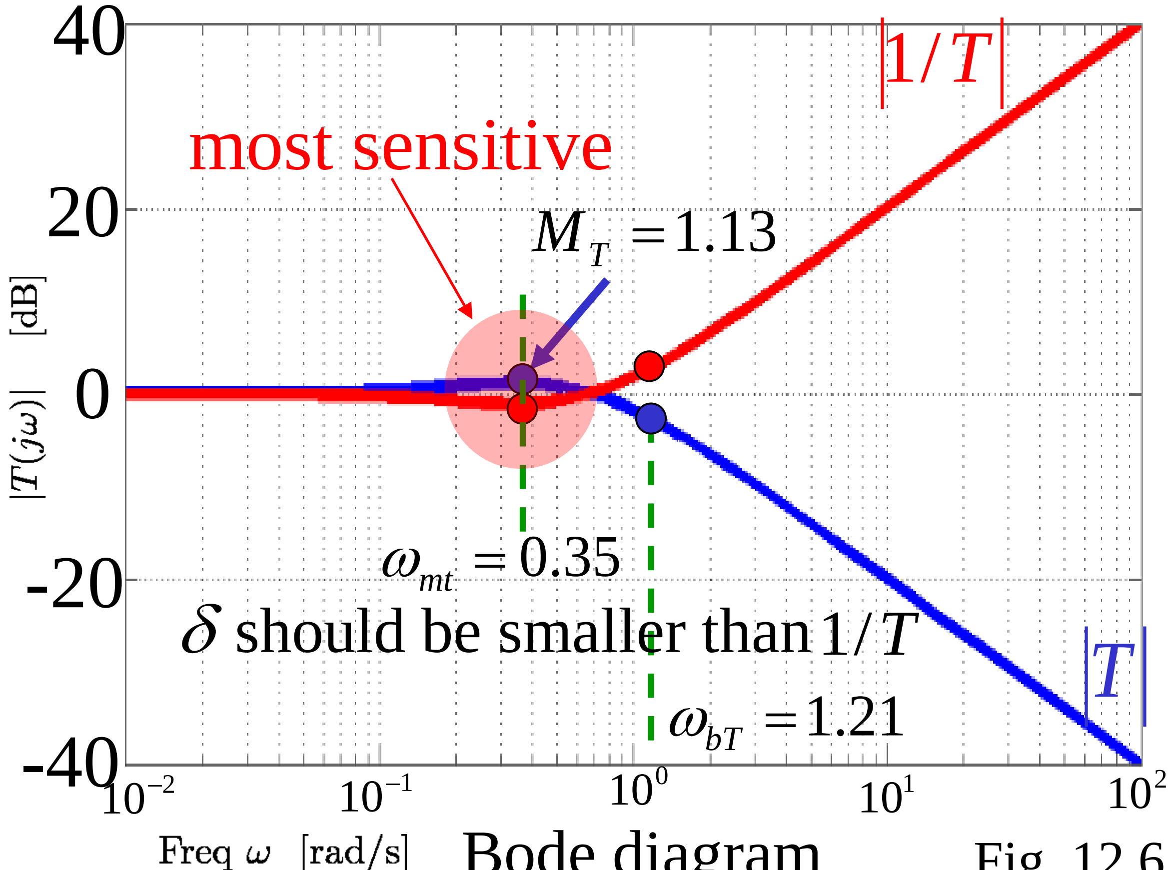
[Ex. 12.5] Cruise Control

Robust Stability (sufficient condition)

$$|\delta(j\omega)| < \frac{1}{|T(j\omega)|}$$

$$P(s) = \frac{1.38}{s + 0.0142}$$

$$C(s) = 0.72 + \frac{0.18}{s}$$: PI controller



Bode diagram Fig. 12.6

[Ex. 12.5] Cruise Control

Robust Stability (sufficient condition)

$$|\delta| < \frac{1}{|T|}$$

around the gain crossover frequencies

small $|\delta(j\omega)|$ is required

⬇

A simple model that describes the process dynamics well around the crossover frequency is often sufficient for design

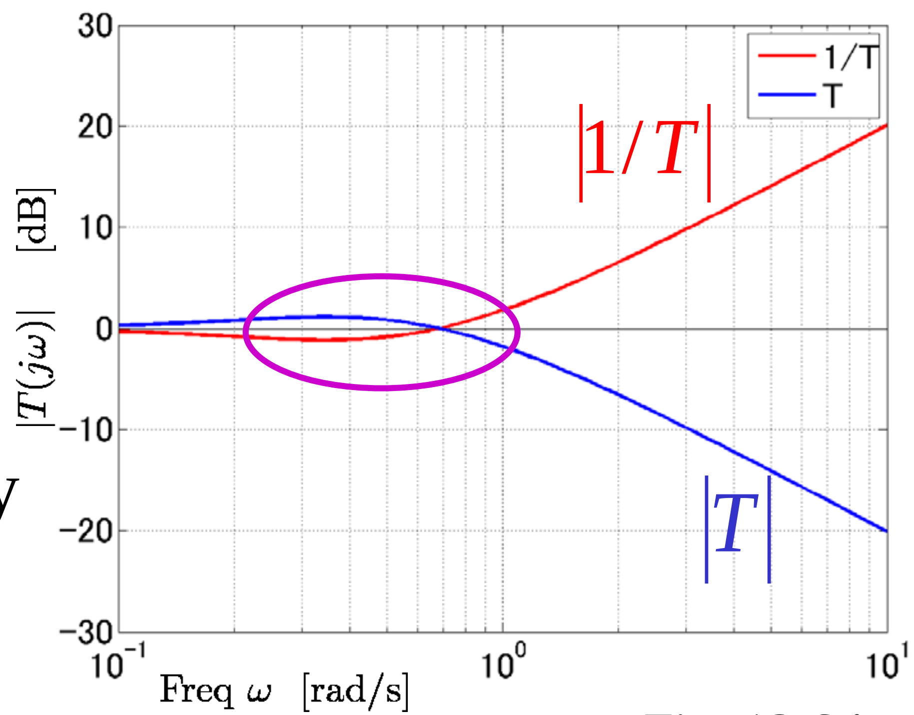


Fig. 12.6 ’

Bode diagram

Robust Stability Using Small Gain Theorem

sufficient condition for robust stability

$$|\delta(j\omega)| < \frac{1}{|T(j\omega)|} \quad \forall \omega \geq 0 \qquad (12.6)$$

another interpretation by using small gain theorem

Theorem 9.4 **Small Gain Theorem** (§ 9.5)

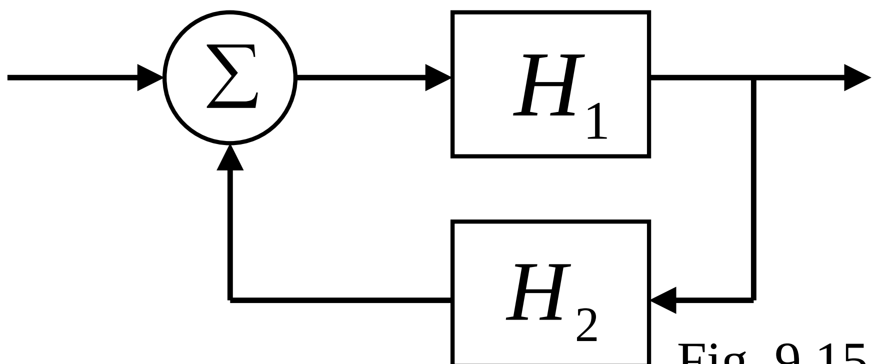


Fig. 9.15

Consider the closed loop system shown in Fig. 9.15, where H_1 and H_2 are stable systems and the signal spaces are properly defined. Let the gains of the systems H_1 and H_2 be γ_1 and γ_2. Then the closed loop system is input/output stable if $\gamma_1\gamma_2 < 1$.

Robust Stability Using Small Gain Theorem

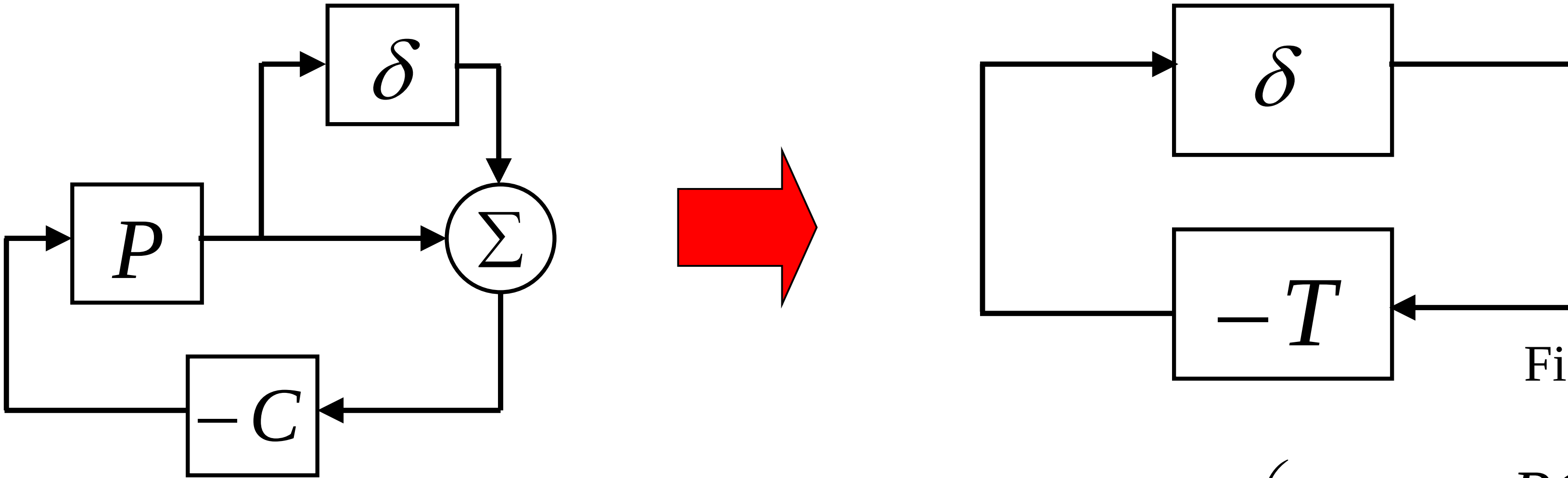


Fig. 12.7

$$\left(\because \; T = \frac{PC}{1+PC} \right)$$

sufficient condition for robust stability

$$|\delta| < \frac{1}{|T|}$$

↓

$$|\delta| \cdot |T| < 1$$

↓ Small Gain Theorem

Closed loop system is input/output stable

Load Sensitivity Function* (§ 11.3)

$$G_{yd} = \frac{P}{1+PC} = PS = \frac{T}{C}$$

$C(s)$: with integral action $\frac{k_i}{s}$

(11.8, 11.9)

$$G_{yd} = \frac{T}{C} \approx \frac{1}{C} \approx \frac{s}{k_i} \text{ for small } \omega \quad (T \approx 1)$$

$$G_{yd} = PS \approx P \qquad \text{for large } \omega \quad (S \approx 1)$$

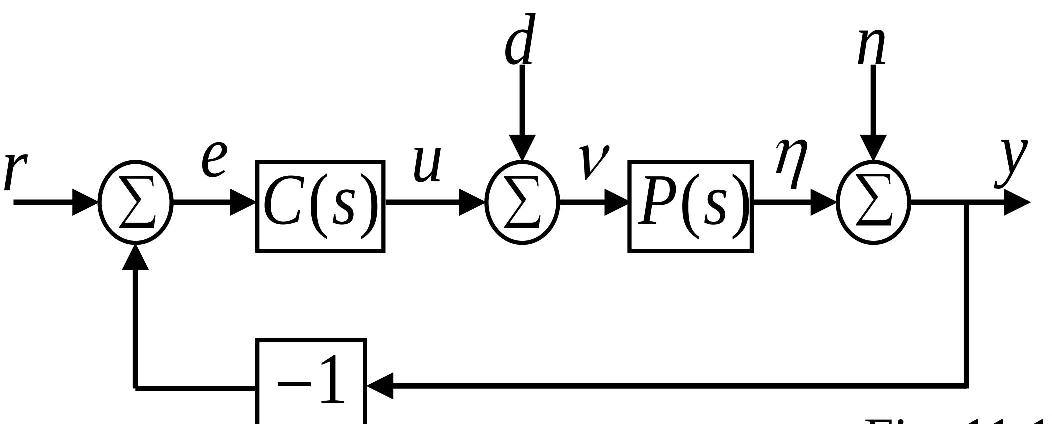
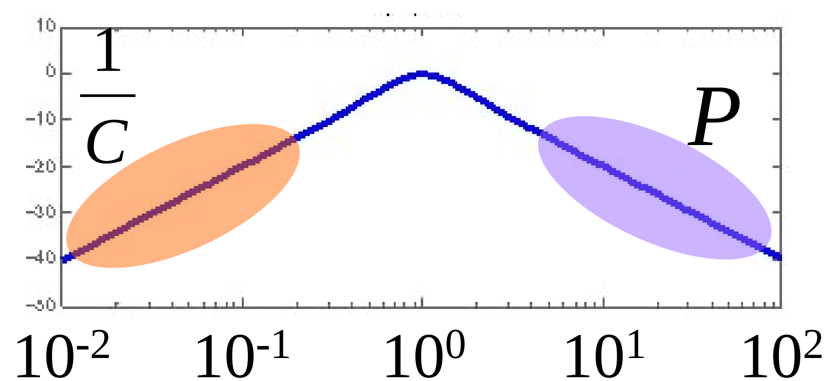


Fig. 11.1



$$P = \frac{1}{s+1}$$

Noise Sensitivity Function* (§ 11.3)

$$-G_{un} = \frac{C}{1+PC} = CS = \frac{T}{P} \quad (11.10)$$

$$-G_{un} = \frac{T}{P} \approx \frac{1}{P} \text{ for small } \omega \quad (T \approx 1)$$

$$-G_{un} = CS \approx C \text{ for large } \omega \quad (S \approx 1)$$

$$P = \frac{1}{(s+1)^3}$$

$$C = \left(k_p + \frac{k_i}{s} + k_d s\right)\frac{1}{T_f^2/2s^2 + T_f s + 1}$$

PID + 2nd-order noise filter

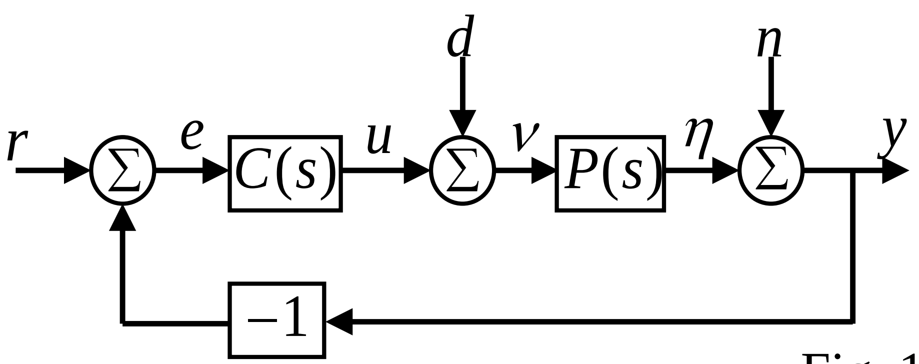
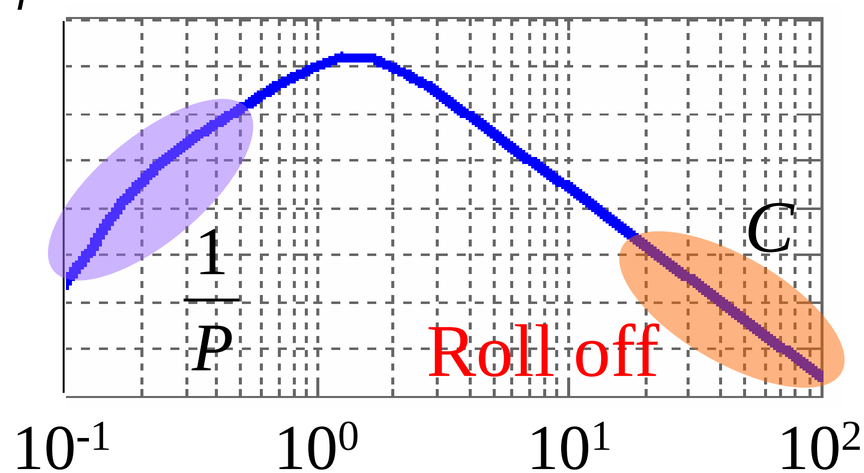


Fig. 11.1



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(12.3 Performance in the Presence of Uncertainty) (pp.358 to 361)

(11.5 Fundamental Limitation) (pp.331 to 340)

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